

DETAILED THERMODYNAMIC ANALYSIS OF FUEL CELL EFFICIENCY

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ABSTRACT

Efficiency of hydrogen fuel cells is analyzed using a non-equilibrium theory of mixtures based on classical irreversible thermodynamics. The efficiency is expressed in terms of processes taking place inside the fuel cells revealing which processes are responsible for efficiency losses. This provides a new method of optimization. It is shown that efficiency losses are not only given by entropy production rate but also by some additional terms, which become important if steep gradients of temperature are present. Consequently, we compare the new theory with the standard entropy production minimization approach. Finally, we discuss effects of the additional terms in polymer electrolyte membrane fuel cells and in solid oxide fuel cells showing that the new theory gives the same results as the standard theory in the former case while it becomes important in the latter case.

1 INTRODUCTION

In this work we present a method of evaluation of fuel cell efficiency which is alternative to the approach of entropy production rate minimization based on the Gouy-Stodola theorem, see Refs. [4, 13, 34, 19]. The method has already been published in Ref. [28], and it is, from a point of view, more suitable for fuel cells than the standard approach, see section 4.

Our analysis is formulated within classical irreversible thermodynamics (CIT), which is very well presented for example in Refs. [29, 10, 24, 17, 18]. CIT has already proven very useful in fuel cell modelling. For example, Kjelstrup and Røsjorde developed in article [20] a comprehensive non-isothermal model of polymer electrolyte membrane fuel cells (PEMFC) based on CIT, where even coupling effects between heat flux and diffusion fluxes were taken into account. Sciacovelli used the method of entropy production rate minimization to optimize a solid oxide fuel cell (SOFC), see Refs. [31, 32].

2 CLASSICAL IRREVERSIBLE THERMODYNAMICS

The analysis of efficiency is formulated within the framework of CIT. Since the system we consider is composed of multiple species, it is necessary to use a theory of mixtures. Many theories of mixtures have been published so far, see Refs. [10, 35, 26, 36, 6, 8, 15, 7, 25, 21, 16, 17, 28]. We chose the theory presented by Jou et al. in chapter 1 of book [17] in the form introduced in paper [28]. We also have to mention that the theory is quite limited since it can be rigorously justified only for a mixture of fluids in local thermodynamic equilibrium while various parts of fuel cells are usually porous media. However, Lebon et al. in article [23] and del Río et al. in article [11] showed that porous media can be described by means of Extended Irreversible Thermodynamics (EIT), see Ref. [17],

which can be seen as a direct extension of CIT. Therefore, we take the theory of mixtures formulated within CIT as the first approximation of a more complex model formulated within EIT. However, probably the most promising approach is the framework of GENERIC, see Refs. [12, 14, 27], since it is capable to take into account the complexity of multi-level description necessary in modeling mixtures containing polymers, see for example Ref. [1]. Let us now turn to the theory of mixtures itself.

Balance of mass can be formulated as

$$\frac{\partial \rho_\alpha}{\partial t} = -\text{div}(\rho_\alpha \mathbf{v}_\alpha) + \hat{\rho}_\alpha, \quad \sum_\alpha \hat{\rho}_\alpha = 0 \quad (1)$$

where ρ_α , \mathbf{v}_α and $\hat{\rho}_\alpha$ are density, velocity and production of species α , respectively. Mass fractions are denoted by $w_\alpha = \frac{\rho_\alpha}{\rho}$ where ρ is the total density $\rho = \sum_\alpha \rho_\alpha$. Diffusive velocity and diffusive flux of the species are defined as

$$\mathbf{v}_{D\alpha} = \mathbf{v}_\alpha - \mathbf{v}, \quad \mathbf{j}_{D\alpha} = \rho_\alpha \mathbf{v}_{D\alpha} \quad (2)$$

respectively where \mathbf{v} stands for the center of mass or barycentric velocity $\mathbf{v} = \sum_\alpha \mathbf{v}_\alpha w_\alpha$.

Balance of linear momentum of species α can be formulated as

$$\frac{\partial \rho_\alpha \mathbf{v}_\alpha}{\partial t} = -\text{div}(\rho_\alpha \mathbf{v}_\alpha \otimes \mathbf{v}_\alpha) + \text{div} \mathbf{t}_\alpha^T + \rho_\alpha \mathbf{F}_\alpha + \hat{\rho}_\alpha \quad (3)$$

where \mathbf{t}_α , \mathbf{F}_α and $\hat{\rho}_\alpha$ stand for partial Cauchy stress tensor of species α , external force exerted on species α and internal transfer of momentum from other species, also called momentum

production rate, respectively. Since no momentum is created by interactions between species, we have $\sum_{\alpha} \hat{p}_{\alpha} = 0$. We suppose that external forces are potential, i.e. $\mathbf{F}_{\alpha} = -\nabla\varphi_{\alpha}$. The total Cauchy stress tensor and the total force are defined as

$$\mathbf{t} = \sum_{\alpha} \mathbf{t}_{\alpha} - \sum_{\alpha} \rho_{\alpha} \mathbf{v}_{D\alpha} \otimes \mathbf{v}_{D\alpha}, \quad \mathbf{F} = \sum_{\alpha} w_{\alpha} \mathbf{F}_{\alpha} \quad (4)$$

respectively. Partial Cauchy stress tensors are assumed to be symmetric, i.e. $\mathbf{t}_{\alpha}^T = \mathbf{t}_{\alpha}$. Dissipative part of the Cauchy stress tensor is defined as $\mathbf{t}_{dis} = \mathbf{t} + p\mathbf{I}$ where p is pressure.

Balance of total energy can be formulated as

$$\begin{aligned} \frac{\partial}{\partial t} \sum_{\alpha} \rho_{\alpha} \left(\frac{\mathbf{v}_{\alpha}^2}{2} + \varepsilon_{\alpha} + \varphi_{\alpha} \right) = \\ - \operatorname{div} \left(\sum_{\alpha} \rho_{\alpha} \mathbf{v}_{\alpha} \left(\frac{\mathbf{v}_{\alpha}^2}{2} + \varepsilon_{\alpha} + \varphi_{\alpha} \right) \right) - \\ - \operatorname{div} \left(\mathbf{j}_q - \sum_{\alpha} \mathbf{t}_{\alpha}^T \cdot \mathbf{v}_{\alpha} \right) + \sum_{\alpha} \rho_{\alpha} \frac{\partial \varphi_{\alpha}}{\partial t} \end{aligned} \quad (5)$$

where ε_{α} is internal energy of species α in the frame of reference of the species itself. The last term in the latter equation represents the change of energy of the material point caused by variation of external force fields. Flux of total energy is then

$$\mathbf{j}_{en} = \mathbf{j}_q + \sum_{\alpha} \rho_{\alpha} \mathbf{v}_{\alpha} \cdot \left(\left(\frac{1}{2} \mathbf{v}_{\alpha}^2 + \varepsilon_{\alpha} + \varphi_{\alpha} \right) \mathbf{I} - \frac{\mathbf{t}_{\alpha}}{\rho_{\alpha}} \right) \quad (6)$$

Internal energy is defined as

$$\rho u = \sum_{\alpha} \rho_{\alpha} \varepsilon_{\alpha} - \frac{1}{2} \rho \mathbf{v}^2 - \sum_{\alpha} \rho_{\alpha} \varphi_{\alpha} \quad (7)$$

and it's balance equation can be obtained straightforwardly from the previous balance equations, see Ref. [28] for details.

Gibbs relation can be expressed as

$$\frac{Ds}{Dt} = \frac{1}{T} \frac{Du}{Dt} + \frac{p}{T} \frac{D}{Dt} \frac{1}{\rho} - \sum_{\alpha} \frac{\mu_{\alpha}}{T} \frac{Dw_{\alpha}}{Dt} \quad (8)$$

where temperature, pressure and chemical potentials were identified as the corresponding derivatives of local equilibrium entropy.

Combining Eq. (8), balance of internal energy and some equilibrium thermodynamic relations following from the assumption of local thermodynamic equilibrium, see Ref. [28] for details, we arrive at the balance of entropy

$$\frac{\partial \rho s}{\partial t} = -\operatorname{div} \mathbf{j}_{s,tot} + \sigma_s \quad (9)$$

where entropy flux and entropy production rate are

$$\mathbf{j}_{s,tot} = \rho s \mathbf{v} + \mathbf{j}_s \quad (10)$$

$$\mathbf{j}_s = \frac{\mathbf{j}_q}{T} + \sum_{\alpha} \mathbf{j}_{D\alpha} s_{\alpha} - \frac{1}{T} \sum_{\alpha} (\mathbf{t}_{\alpha}^T + p v_{\alpha} \rho_{\alpha} \mathbf{I}) \cdot \mathbf{v}_{D\alpha} \quad (11)$$

$$\begin{aligned} \sigma_s = & -\frac{1}{T} \mathbf{j}_s \cdot \nabla T - \sum_{\alpha} \frac{1}{T} \mathbf{j}_{D\alpha} \cdot \nabla \tilde{\mu}_{\alpha} + \\ & + \frac{1}{T} \mathbf{t}_{dis} : \mathbf{d} + \sum_r \frac{1}{T} \dot{\xi}_r \tilde{A}_r \end{aligned} \quad (12)$$

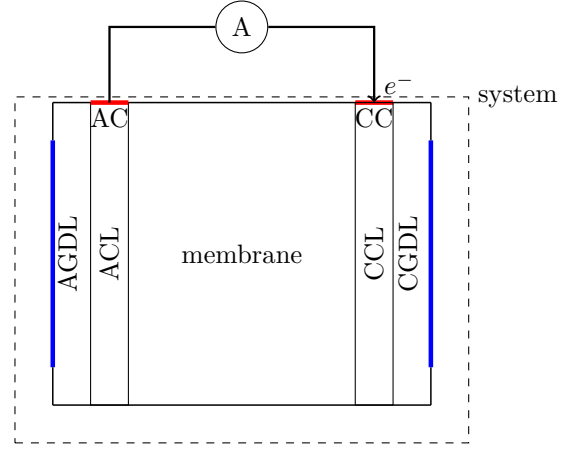


Figure 1. A scheme of a PEM fuel cell for better understanding of the acronyms.

where $\tilde{\mu}_{\alpha}$ and \tilde{A}_r are electrochemical potential of species α and electrochemical affinity of reaction r , respectively, defined as

$$\tilde{\mu}_{\alpha} = \mu_{\alpha} + \varphi_{\alpha} \quad \tilde{A}_r = -\sum_{\alpha} v_{\alpha r} \tilde{\mu}_{\alpha} \quad (13)$$

where $v_{\alpha r}$ is the stoichiometric coefficient of species α in reaction r . $\dot{\xi}_r$ in Eq. (12) is the rate of reaction r . The second law of thermodynamics is expressed as

$$\sigma_s \geq 0 \quad (14)$$

For a mixture of ideal gases subject to external forces and for a single-species non-viscous continuum the total flux of energy (6) can be simplified to

$$\mathbf{j}_{en} = \mathbf{j}_q + \sum_{\alpha} \rho_{\alpha} \mathbf{v}_{\alpha} \left(\frac{1}{2} \mathbf{v}_{\alpha}^2 + h_{\alpha} + \varphi_{\alpha} \right) \quad (15)$$

where h_{α} is partial specific enthalpy¹. Moreover, for such mixtures entropy flux (11) simplifies to

$$\mathbf{j}_s = \frac{\mathbf{j}_q}{T} + \sum_{\alpha} \mathbf{j}_{D\alpha} s_{\alpha} \quad (16)$$

From the assumption of local thermodynamic equilibrium it follows that

$$h_{\alpha} = \left(\frac{\partial H}{\partial m_{\alpha}} \right)_{T,p,m_{\beta \neq \alpha}} = u_{\alpha} + p v_{\alpha} = T s_{\alpha} + \mu_{\alpha} \quad (17)$$

3 EFFICIENCY OF FUEL CELLS

For clarity of terminology the thermodynamic analysis is formulated for PEMFC. The results, however, hold also for other kinds of fuel cells, e.g. SOFC. In figure 1 we show a scheme of a PEM fuel cell. The fuel cell is composed of the following parts: anode gas diffusion layer (AGDL), through which

¹Of course, in case of single-species continuum the summation is not present.

hydrogen and water flow, cathode gas diffusion layer (CGDL), through which oxygen and water flow, an anode, where hydrogen splits into protons and electrons, a cathode, where oxygen reacts with protons and electrons forming water, and a membrane, through which protons are transported from the anode to the cathode. Hereafter, these parts together, represented by the area enclosed in the dashed frame in figure 1, are referred to as the system. The system is denoted by V and its boundary by ∂V . We assume that the fuel cell is in a stationary state.

In order to evaluate efficiency of the fuel cell, fluxes of energy and entropy into and out of the system are analyzed. Energy and entropy flow into and out of the system through AGDL, CGDL, through the conductive part of the anode (CA) from which electrons leave the system and through the conductive part of the cathode (CC) where electrons re-enter the system.

Fluxes of enthalpy, Gibbs energy and conductive heat into the system are defined as

$$\Delta H = - \int_{\partial V} \sum_{\alpha \neq e^-} h_{\alpha} \rho_{\alpha} \mathbf{v}_{\alpha} \cdot d\mathbf{S} \quad (18)$$

$$\Delta G = - \int_{\partial V} \sum_{\alpha \neq e^-} \mu_{\alpha} \rho_{\alpha} \mathbf{v}_{\alpha} \cdot d\mathbf{S} \quad (19)$$

$$\Delta Q = - \int_{\partial V} \mathbf{j}_q \cdot d\mathbf{S} \quad (20)$$

respectively. The fluxes are positive when flowing into the system. Note that electrons are excluded from the fluxes. They are analyzed separately since they are closely related to electrical work.

Electrical work extracted from the fuel cell ΔW_{el} is the flux of total energy carried by electrons through boundary of the system, i.e. through AC and CC. Therefore, from Eqs. (15) and (17) it follows² that the electrical work can be expressed as

$$\begin{aligned} \Delta W_{el} = & \frac{I}{F} M_{e^-} (\mu_{e^-}^a - \mu_{e^-}^c) + I(\Phi^c - \Phi^a) + \\ & + \frac{I}{F} (T^a s_{e^-}^a - T^c s_{e^-}^c) \end{aligned} \quad (21)$$

where I denotes the total electric current flowing from the cathode to the anode. Obviously, the first two terms on the right hand side of the latter equation represent the standard electrical work. The last term is related to thermoelectric power caused by possibly different temperatures at AC and CC. However, it was shown in Ref. [28] that the term is usually quite negligible.

Stationary balance of total energy³ gives

$$0 = \int_{\partial V} \mathbf{j}_{en} \cdot d\mathbf{S} \quad (22)$$

Using definitions (18), (20) and (21) and expression for energy flux in Eq. (15), we obtain

$$\Delta W_{el} = \Delta Q + \Delta H \quad (23)$$

which is a well known formula, used for example in Ref. [3].

In the stationary state local balance of entropy (9) gives

$$\text{div} \mathbf{j}_{s,tot} = \sigma_s \quad (24)$$

Flux of entropy can be generally expressed, according to Eq. (11), as

$$\mathbf{j}_s = \frac{1}{T} \mathbf{j}_q + \sum_{\alpha} s_{\alpha} \mathbf{j}_{D\alpha} + \mathbf{j}'_s \quad (25)$$

where \mathbf{j}'_s disappears at ∂V in our case. Therefore, Eq. (16) describes entropy flux at the boundary. Hence, we get for the flux of heat into the system

$$\begin{aligned} \Delta Q = & \int_{\partial V} \left(-T \mathbf{j}_{s,tot} + T \sum_{\alpha} \rho_{\alpha} \mathbf{v}_{\alpha} s_{\alpha} \right) \cdot d\mathbf{S} \\ = & \Delta G - \Delta H + \int_{\partial V} -T \mathbf{j}_{s,tot} \cdot d\mathbf{S} + \\ & + \frac{I}{F} (T^a s_{e^-}^a - T^c s_{e^-}^c) \end{aligned} \quad (26)$$

where Eq. (17) was used. From the stationary balance of entropy (24) we get

$$\begin{aligned} \int_{\partial V} -T \mathbf{j}_{s,tot} \cdot d\mathbf{S} = & - \int_V \text{div}(T \mathbf{j}_{s,tot}) dV = \\ = & \int_V -\mathbf{j}_{s,tot} \cdot \nabla T dV - \Pi \end{aligned} \quad (27)$$

where dissipation in the system was introduced as

$$\Pi = \int_V T \sigma_s dV \quad (28)$$

Using relations (23), (26) and (27), we obtain the final formula for electrical work

$$\begin{aligned} \Delta W_{el} = & \Delta G - \Pi + \frac{I}{F} M_{e^-} (T^a s_{e^-}^a - T^c s_{e^-}^c) - \\ & - \int_V \mathbf{j}_{s,tot} \cdot \nabla T dV \end{aligned} \quad (29)$$

The first term on the right hand side of the latter equation expresses that electrical work is produced from flux of Gibbs energy into the system. The second term expresses that entropy production rate inevitably lowers the electrical work produced. The next to last term shows a direct contribution of thermoelectric effects and the last term contains dissipation due to temperature gradient, see the formula for entropy production rate (12),

$$\int_V -\mathbf{j}_s \cdot \nabla T dV \quad (30)$$

Therefore, this part of entropy production rate does not reduce the produced electrical work since it cancels with the same term in the dissipation term.

Efficiency of fuel cells is defined as, see for example Ref. [3],

$$\eta = \frac{\Delta W_{el}}{\Delta H} \quad (31)$$

²We assume that electrons can be described with the electron gas approximation, see §57 in book [22].

³We neglect kinetic energy since it is negligible compared to other kinds of energy flowing into the system.

Therefore, using the formula for electrical work (29), we obtain

$$\begin{aligned}
\eta &= \frac{\Delta G}{\Delta H} \frac{\Delta W_{el}}{\Delta G} = \\
&= \eta_c \left(1 - \frac{\Pi}{\Delta G} \right) - \frac{\eta_c}{\Delta G} (T^c s_{e^-}^c - T^a s_{e^-}^a) \frac{I}{F} \\
&\quad - \frac{\eta_c}{\Delta G} \int_V \mathbf{j}_{s,tot} \cdot \nabla T dV = \\
&= \eta_c \left(1 - \eta_{e^-} - \frac{1}{\Delta G} \int_V \left(\rho_{sv} \cdot \nabla T - \right. \right. \\
&\quad \left. \left. - \sum_{\alpha} \mathbf{j}_{D\alpha} \cdot \nabla \tilde{\mu}_{\alpha} + \mathbf{t}_{dis} : \mathbf{d} + \sum_r \xi_r \tilde{A}_r \right) dV \right) \quad (32)
\end{aligned}$$

where the conversion efficiency and change of efficiency due to entropy of electrons were defined as

$$\eta_c = \frac{\Delta G}{\Delta H}, \quad \eta_{e^-} = \frac{(T^a s_{e^-}^a - T^c s_{e^-}^c) \frac{I}{F}}{\Delta G} \quad (34)$$

respectively.

Conversion efficiency η_c can be calculated from definitions (19) and (18) using thermodynamic tables, see for example book [33]. It can be approximated as 80% at 80°C and standard pressure as shown in Barbir's book [3]. The last three terms in Eq. (32) are not standard and they are referred to as the new terms later in this work. Their effects on efficiency are discussed in section 5.

The second law of thermodynamics, see Eq. (14), dictates that $\Pi \geq 0$. Therefore, dissipation inevitably reduces efficiency of fuel cells.

4 COMPARISON OF THE STANDARD AND THE PROPOSED THEORY

The standard theory is based on the Gouy-Stodola theorem as has been already said in the introduction. The purpose of this section is to clarify the possible advantages of the new theory.

What does optimization of fuel cells mean, in fact? Recalling the definition of efficiency (31), the optimization is a way of maximization of the electrical work obtained from conversion of the fuel to the exhausts, i.e. in hydrogen fuel cells from combustion of hydrogen. From the thermodynamic point of view, the conversion can be characterized by ΔH and ΔG . Therefore, the optimization means maximizing the gained electrical work for given ΔH and ΔG or, recalling Eq. (32), maximizing η keeping ΔH and ΔG constant.

Let us now have a closer look at formula (32). Obviously, entropy production rate reduces the efficiency since if it were zero everywhere in the system, the dissipation term, which is always non-negative, would disappear. This is a common point of both the standard and the new theory.

However, as has been said above the optimization means maximizing η while keeping ΔG and η_c constant⁴. If we set the dissipation to zero, there is still the last term in (32), which affects the efficiency. Hence, the efficiency could be further enhanced by optimization of this term. Moreover, the same holds for the next to last term in Eq. (32) as well but since it is negligible, we do not take it into account.

Hence, from Eq. (33) it follows that if we neglect the η_{e^-} term and the term with barycentric velocity, we should minimize the functional

$$\int_V - \sum_{\alpha} \mathbf{j}_{D\alpha} \cdot \nabla \tilde{\mu}_{\alpha} + \mathbf{t}_{dis} : \mathbf{d} + \sum_r \xi_r \tilde{A}_r dV \quad (35)$$

rather than entropy production rate itself.

Of course, this functional should be understood as the first approximation of a more precise expression since we are restricted to the area of validity of CIT and the theory of mixtures formulated within. More microscopic levels of description should be, therefore, taken into account to obtain more accurate results for the efficiency. Hence, optimization of fuel cells is an area where a more precise framework, for example the GENERIC framework in Ref. [1], is needed.

5 POSSIBLE ENHANCEMENTS OF EFFICIENCY

In this section we analyze influence of the new terms on efficiency of PEMFC and SOFC. To avoid repetition we only summarize results of a previous analysis provided in Ref. [28], where details can be found. Estimates of effects of the new terms in Eq. (32) on efficiency of fuel cells are based on the thermodynamic analysis of Kjelstrup and Røsjorde in article [20]. We also evaluate the effects for a concrete set of conditions considered in the paper. For SOFC our analysis is based on works of Sciacovelli [31, 32].

5.1 PEMFC

The first new term in formula for efficiency (32)

$$- \eta_c \frac{\frac{I}{F} (T^c s_{e^-}^c - T^a s_{e^-}^a)}{\Delta G} \quad (36)$$

describes contribution of entropy of electrons transported along when electric current leaving and re-entering the fuel cell. It's contribution to the final efficiency is, however, negligible⁵ as has been shown in Ref. [28].

The second new term⁶ in (32) is

$$- \frac{\int_V \frac{\mathbf{j}_q}{T} \cdot \nabla T dV}{\Delta G} - \frac{\int_V \sum_{\alpha} \rho_{\alpha} s_{\alpha} \mathbf{v}_{\alpha} \cdot \nabla T dV}{\Delta G} \quad (37)$$

Conductive heat flux affects the efficiency by 0.2%. Calculating partial specific entropy of gaseous oxygen, hydrogen, water and protons at 80°C, the effect of flux of entropy in gradient of temperature in Eq. (37) to the efficiency can be estimated as 0.1%, see Ref. [28] for details.

In summary, the new terms in (32) are quite negligible in PEM fuel cells compared to the dissipation term. Therefore, entropy production rate is the main cause of efficiency losses in PEM fuel cells and the new method and the standard method are equivalent in this case. This is not true for SOFC anymore as we show in section 5.2.

⁴Recall that η_c is a function of ΔG and ΔH , see Eq. (34).

⁵approximately 10⁻³%

⁶For simplicity, the additional entropy flux \mathbf{j}'_s is neglected.

5.2 SOFC

In this section we summarize effects of the new terms in Eq. (32) on efficiency of SOFC. Generally, the effect is higher than in PEMFC because temperature gradients are steeper in SOFC. Thermodynamic quantities will be evaluated at temperature 880°C to correspond with [31]. At that temperature the corresponding conversion efficiency becomes approximately $\eta_c \approx 74\%$.

Usually, the biggest part of a SOFC is the anode. Let us now summarize effects of the new terms in the anode. Contribution of oxygen to the last term in Eq. (37) was estimated as 2%. The analogical contribution of hydrogen was estimated as 3% and the contribution of water vapor as 4%. Moreover, oxygen anions diffuse through the solid part of the anode. Partial molar entropy of oxygen anions in the anode was estimated using Refs. [9, 2, 18, 30]. Effect of oxygen anions on the efficiency was then estimated as 0.7%. The effect of conductive heat flux in Eq. (37) was estimated using Refs. [5, 31] as 0.1%. See Ref. [28] for details.

As well as in PEMFC the effect of entropy of electrons η_{e^-} is negligible in SOFC.

In summary, the new terms in (32) affect the total efficiency by several per cent. Therefore, the new method of evaluation of fuel cell efficiency differs from the standard method in case of SOFC, and we suggest that the new method should be used instead of the standard method in that case, i.e. functional (35) should be minimized instead of entropy production rate.

6 CONCLUSIONS

In this work we discussed the meaning of optimization of hydrogen fuel cells, see section 4. A formula for efficiency compatible with the meaning of optimization, which has been derived in Ref. [28] using classical irreversible thermodynamics of mixtures, was presented in sections 2 and 3. The formula was then compared with the standard theory based on the Gouy-Stodola theorem, which leads to the procedure of entropy production rate minimization.

The new theory gives different results than the standard theory in case of solid oxide fuel cells while it is equivalent to the standard theory in case of polymer electrolyte membrane fuel cells, see section 5. In summary, we propose that the new theory should be used in optimization of solid oxide fuel cells instead of the standard theory. In other words, functional (35) should be minimized instead of entropy production rate in the fuel cells.

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NOMENCLATURE

A cross section area of the anode [m^2]
 \tilde{A}_r electrochemical affinity of reaction r [J kg^{-1}]
 \mathbf{d} symmetric velocity gradient [s^{-1}]
 \mathbf{F} total external body force [N kg^{-1}]

F Faraday constant [C mol^{-1}]
 \mathbf{F}_α external body force exerted on species α [N kg^{-1}]
 h_α partial specific enthalpy [J kg^{-1}]
 I total electric current passing through the system [A]
 $\mathbf{j}_{D\alpha}$ diffusion flux of species α [$\text{kg m}^{-2} \text{s}^{-1}$]
 \mathbf{j}_{en} flux of total energy [$\text{J m}^{-2} \text{s}^{-1}$]
 \mathbf{j}_s flux of entropy [$\text{JK}^{-1} \text{m}^{-2} \text{s}^{-1}$]
 $\mathbf{j}_{s,tot}$ total flux of entropy [$\text{JK}^{-1} \text{m}^{-2} \text{s}^{-1}$]
 \mathbf{j}_q conductive heat flux [$\text{J m}^{-2} \text{s}^{-1}$]
 $\frac{D_\bullet}{Dt}$ material time derivative [s^{-1}]
 p pressure [Pa]
 p_α partial pressure of species α [Pa]
 \hat{p}_α production of momentum of species α due to interactions with other species [$\text{kg m}^{-2} \text{s}^{-2}$]
 s specific entropy [$\text{JK}^{-1} \text{kg}^{-1}$]
 s_α partial specific entropy of species α [$\text{J kg}^{-1} \text{K}^{-1}$]
 $s_{e^-}^a$ partial specific entropy of electrons in the anode [$\text{JK}^{-1} \text{mol}^{-1}$]
 $s_{e^-}^c$ partial specific entropy of electrons in the cathode [$\text{JK}^{-1} \text{mol}^{-1}$]
 V total volume [m^3]
 \mathbf{v}_α velocity of species α [m s^{-1}]
 \mathbf{v} barycentric velocity [m s^{-1}]
 v_α partial specific volume of species α [$\text{m}^3 \text{kg}^{-1}$]
 $\mathbf{v}_{D\alpha}$ diffusive velocity of species α [m s^{-1}]
 T temperature [K]
 \mathbf{t} total Cauchy stress tensor [J m^{-3}]
 \mathbf{t}_{dis} dissipative part of the Cauchy stress tensor [Pa]
 \mathbf{t}_α partial Cauchy stress tensor of species α [J m^{-3}]
 u total specific internal energy [J kg^{-1}]
 u_α partial specific internal energy of species α [J kg^{-1}]
 w_α mass fraction of species α [1]
 ΔG total flux of Gibbs free energy into the system [J s^{-1}]
 ΔH total flux of enthalpy into the system [J s^{-1}]
 ΔQ total flux of heat into the system [J s^{-1}]
 ΔW_{el} electrical work extracted from the system [J s^{-1}]
 ε_α specific internal energy of species α with respect to its own frame of reference [J kg^{-1}]
 ε_F Fermi energy [J]
 η total efficiency [1]
 η_c conversion efficiency [1]
 η_{e^-} change of efficiency due to entropy of electrons [1]
 μ_α chemical potential of species α [J kg^{-1}]
 $\tilde{\mu}_\alpha$ electrochemical potential of species α [J kg^{-1}]
 $\nu_{\alpha r}$ stoichiometric coefficient of species α in reaction r [1]
 ξ_r rate of chemical reaction r [$\text{kg m}^{-3} \text{s}^{-1}$]
 Π total dissipation [J s^{-1}]
 ρ overall density [kg m^{-3}]
 ρ_α density of species α [kg m^{-3}]
 $\hat{\rho}_\alpha$ production of species α [$\text{kg s}^{-1} \text{m}^{-3}$]
 \sum_α sum over all species
 σ_s entropy production rate [$\text{JK}^{-1} \text{m}^{-3} \text{s}^{-1}$]
 Φ electric potential [V]
 φ_α potential of external force \mathbf{F}_α [J kg^{-1}]
 χ_g characteristic function of gas phase [1]
 χ_l characteristic function of liquid phase [1]

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