

THE INFEASIBILITY OF REVERSIBILITY AND ITS CONSEQUENCES FOR ENGINEERING DESIGN

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ABSTRACT

Applications like exergy and entropy generation minimization (EGM) are widely used in engineering research and industry. Exergy attributes work potential to heat and therefore it allows to conduct meaningful analyses on systems where the First Law seems to fail. Entropy generation minimization, the design methodology used to seize the opportunities identified by exergy analyses, enables the engineer to optimize thermodynamic efficiency of systems under consideration. However, it seems that popularity of the Second Law in engineering has pushed its application beyond the limits. Does the result of an exergy analysis allow to allocate engineering efforts? Can we consider and isolate components or local phenomena in an EGM procedure without fully taking into account their interdependencies? Although those questions appear to be answered affirmative in a significant amount of recent publications, we question the accuracy of that answer in this paper by presenting a number of illustrative examples.

INTRODUCTION

The pursuit of solutions to a problem drives the engineer to apply analysis and design techniques on a system aiming at technical and economic opportunities. One analysis technique which acquired major scientific attention in recent decades is the exergy analysis together with its design counterpart entropy generation minimization (EGM) [1–5]. This paper endeavors to elaborate an assessment on these Second Law based techniques and their use in engineering methodologies.

Exergy analyses pinpoint and quantify thermodynamic imperfections as irreversibilities which either are not identified or misevaluated by energy analyses [6, 7]. These irreversibilities are the differences between the actual work performed and the maximum theoretical useful work obtained if a system is brought into thermodynamic equilibrium [4]. Entropy generation minimization on the other hand seizes the possibilities exergy analyses identify as entropy generation is directly proportional to irreversibility. Minimization of the total entropy generation is therefore equal to system efficiency optimization [8].

In this paper we present the limits of exergy analyses and entropy generation minimization indicated by the infeasibility of reversibility. Illustrative cases are discussed to demonstrate how the inevitability of irreversibility on macro-level crops the applicability of both Second Law based techniques. Although the authors are convinced of the benefits of exergy analyses to visualize losses and of entropy generation minimization as a cost function to optimize an entire system, it seems appropriate to sound a note of caution considering their application on subsystems which are thermodynamically connected.

The relevance of this work can be founded by following non-exhaustive list of articles dealing with Second Law analysis and design on a local scale without considering the system in

which these parts, components or subsystems (eventually) operate [9–17]. Moreover the content of this paper will provide an argument to regard the Second Law of thermodynamics as an alternative for rather than an addition to the First Law in engineering analysis and design.

The remainder of this contribution is structured as follows. We start with a comparison between First and Second Law efficiency and discuss the implications of the differences. Subsequently we offer three perspectives on the Second Law in engineering: a modeling, analysis and design perspective. These perspectives enable us to demarcate the field of application of the Second Law in engineering. Finally conclusions are summarized.

SECOND LAW EFFICIENCY

Efficiency is a ratio of actual performance and ideal performance. The essential difference between First Law efficiency and Second Law efficiency is the definition of that ideal performance which serves as a benchmark. The First Law of thermodynamics puts every form of energy on the same level

$$\frac{\partial E}{\partial t} = \left[\sum \dot{m} \left(h + \frac{1}{2} V^2 + gz \right) \right]_{\text{out}}^{\text{in}} + \sum_{i=0}^n \dot{Q}_i - \dot{W}. \quad (1)$$

From a First Law perspective heat and power are therefore interchangeable modes of energy transfer. The Second Law of thermodynamics associates heat transfer with entropy

$$\frac{\partial S}{\partial t} \geq \sum_{\text{in}} \dot{m} s - \sum_{\text{out}} \dot{m} s + \sum_{i=0}^n \frac{\dot{Q}_i}{T_i}, \quad (2)$$

$$\dot{S}_{\text{gen}} = \frac{\partial S}{\partial t} - \sum_{i=0}^n \frac{\dot{Q}_i}{T_i} - \sum_{\text{in}} \dot{m} s + \sum_{\text{out}} \dot{m} s. \quad (3)$$

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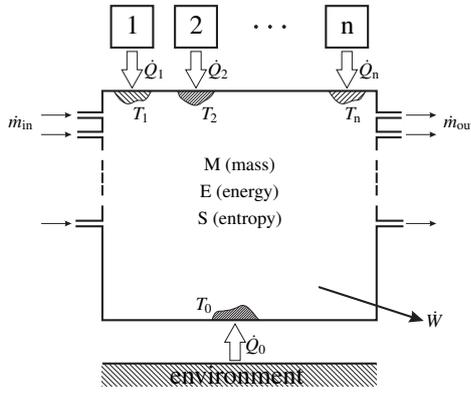


Figure 1. General representation of an open thermodynamic system.

Elimination of \dot{Q}_0 in Eq. (1) and (3) gives the Gouy-Stodola theorem [18]

$$\dot{W} = \left[\sum \dot{m} \left(h - T_0 s + \frac{1}{2} V^2 + gz \right) \right]_{\text{out}}^{\text{in}} - \frac{\partial}{\partial t} (E - T_0 S) \quad (4)$$

$$+ \sum_{i=1}^n \dot{Q}_i \underbrace{\left(1 - \frac{T_0}{T_i} \right)}_{\eta_c} - \underbrace{T_0 \dot{S}_{\text{gen}}}_i$$

which indicates that the Second Law introduces a scaling factor, known as the Carnot efficiency η_c , to devalue heat transfer. Due to this devaluation, the Second Law benchmark will always be lower than the First Law benchmark.

Although the ideal performance associated with the First Law and Second Law are different, optimization of First and Second Law efficiency both lead to the design with maximum thermodynamic efficiency. The benchmark put forward by both thermodynamic laws are in reality unattainable. Whether one minimizes the gap between reality and the First Law ideal or the Second Law ideal, the absolute gap reduction will be equal. Therefore we can state that Second Law efficiency optimization is an alternative for First Law efficiency optimization.

The First Law holds energy conservation and falls short in defining an efficiency metric for components which only transfer (and not use or transform) energy. The Second Law pinpoints all losses \dot{I} including those associated with energy transfer. As such there exists a Second Law efficiency for components like heat sinks and heat exchangers. Therefore Second Law efficiency is a popular objective function to design those type of components (e.g. [9, 14, 16]).

PERSPECTIVES ON THE SECOND LAW

The birth of the Second Law of thermodynamics is associated with the work “Réflexions sur la puissance motrice du feu et sur les machines propre à développer cette puissance” (1824) by Sadi Carnot, a French military engineer and physicist [19]. A few decades later, Lord Kelvin and Clausius formalized the Second Law of thermodynamics. Ever since the Second Law has been of major interest in exact and applied sciences [20].

Today the Second Law has several appearances in engineering. In this section we provide and illustrate three perspectives on the application of the Second Law in engineering, i.e. a modeling perspective, an analysis perspective and a design perspective.

Modeling

A thermodynamic model is a mathematical representation of a physical situation, defined by a system, the system boundary and the environment [21]. A system is a quantity of matter or a region in space upon which attention is concentrated in the analysis of a problem [22]. As such the definition of a system is an artificial concept to isolate scientific focus justifying a model to describe reality. The correspondence of a model to reality however is heavily dependent on the choice of the system boundary which separates the system from the environment and on the mathematical description of the interaction between system and environment.

The exergy method can be regarded as a modeling technique with a peculiar definition of the environment and its interaction with the system. The environment is a very large body or medium in the state of perfect thermodynamic equilibrium. It has no gradients or differences involving pressure, temperature, chemical potential, kinetic or potential energy [23]. The interaction between the system and the environment is represented by a Carnot engine. The work output of this reversible machine is the exergy of the system. It is the maximum theoretical useful work obtained if the system is brought into thermodynamic equilibrium with the environment by means of processes in which the system interacts only with this environment (Gibbs) [4].

Exergy represents a reversible limit which reveals what is impossible rather than what is feasible. The mathematical derivation of exergy only incorporates the equality sign of the Second Law of thermodynamics and with this it omits constraints reality imposes (e.g. time, material properties). Exergy therefore is an inaccurate model to describe reality. The question that arises is: “Can you draw conclusions based on an exergy model?”

Analysis

Second Law analysis comprises a comparison of reality (or an accurate model) with the corresponding exergy model. It uses exergy as a benchmark to pinpoint and quantify thermodynamic imperfection as irreversibility which is the difference between the actual work performed and the maximum theoretical useful work determined by the reversible model (Carnot).

As an example consider a thermal power plant. A power plant generates electricity from mechanical power which is obtained through a conversion of thermal power (\dot{Q} on T_{cc}). Figure 2 shows two possible thermodynamic models for this installation. On the left hand side there is the exergy model which represents the ideal power plant. According to this model the power plant output power is

$$\dot{W}_c = \dot{Q} \left(1 - \frac{T_0}{T_{cc}} \right). \quad (5)$$

On the right hand side we have a more realistic model presented by A. Bejan [3]. This endoreversible power plant model isolates the irreversibility due to heat transfer across finite temperature differences by inserting two heat exchangers (HE1 and HE2) with a limited heat transfer surface inventory ($C \leq C_1 + C_2$). Based on this model the maximum power plant output is

$$\dot{W}_B = \dot{Q} \left(1 - \frac{T_0}{T_{cc} - \dot{Q}/C} \right), \quad (6)$$

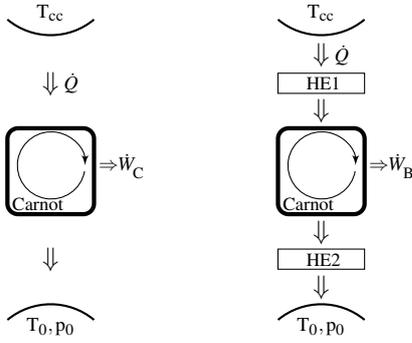


Figure 2. Power plant: exergy model (l), model by Bejan (r) [3].

which is smaller than \dot{W}_C .

An exergy analysis reveals irreversibility as a result of heat transfer across finite temperature differences. Unfortunately this irreversibility can not be eliminated since finite time and space, construction material properties, system topology and economic considerations constrain the heat transfer surface inventory (cf. constraint on C in the endoreversible model). In reality some irreversibilities are intrinsic and consequently unavoidable [18, 23–29]. A quantification of losses based on a comparison with the reversible limit (exergy model) is deceptive because large irreversibilities can be imposed and are therefore inevitable [29]. The value of irreversibility as a result of an exergy analysis does not indicate the potential to reduce it.

An exergy analysis implicitly performs a system decomposition as it aspires to compare reality with the reversible ideal on a local scale to allocate engineering efforts [3]. However, although an exergy analysis pinpoints and quantifies losses which might or might not be reducible, it does not necessarily reveal the source of these losses. Some irreversibilities are caused by the component in which they occur (endogenous exergy destruction) others are caused by other components (exogenous exergy destruction) [27, 28, 30, 31] which implies that a reduction of irreversibility in one component can induce a larger increase of irreversibility in another component [30]. This can be understood by considering the discrepancy between reality and the exergy model since the latter one inherently does not take into account any interaction between interconnected components. A local reduction of irreversibility can therefore have a pernicious effect on the overall efficiency.

As an illustration, consider a heat sink cooled chip (see Fig. 3). The chip provides a heat load \dot{Q} at a junction temperature T_j with a corresponding exergy

$$\dot{E}_{\text{chip}} = \iint_A \dot{q}'' \left(1 - \frac{T_0}{T_j} \right) dA \quad (7)$$

and hands it over to the heat sink.

Subsequently this exergy is (partly) transferred to the passing fluid

$$\Delta \dot{E}_f = \dot{m} (e_{f,\text{out}} - e_{f,\text{in}}). \quad (8)$$

The endogenous irreversibility of the heat sink is then naturally defined as

$$\dot{I}_{\text{hs}} \equiv \dot{E}_{\text{chip}} - \Delta \dot{E}_f. \quad (9)$$

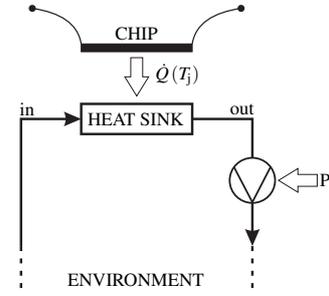


Figure 3. Schematic representation of a heat sink cooled chip.

The chip itself receives electric power and converts it to the heat load \dot{Q} . The conversion of electricity to heat generates an irreversibility \dot{I}_{chip} which can be decomposed in an intrinsic part \dot{I}_i and an avoidable part \dot{I}_a

$$\dot{I}_{\text{chip}} \equiv \dot{Q} - \dot{E}_{\text{chip}}, \quad (10)$$

$$\equiv \dot{I}_i + \dot{I}_a. \quad (11)$$

The intrinsic irreversibility \dot{I}_i is the loss of exergy due to the conversion from electricity to heat at a temperature $T_{j,\text{max}}$. This loss is fixed by electrical integrity of the chip and in particular by the maximum allowable junction temperature $T_{j,\text{max}}$ which is a technical constraint.

$$\dot{I}_i = \iint_A \dot{q}'' \left(\frac{T_0}{T_{j,\text{max}}} \right) dA \quad (12)$$

The avoidable irreversibility \dot{I}_a on the other hand is the loss of exergy due to the fact that the junction temperature T_j remains below the maximum allowable junction temperature $T_{j,\text{max}}$.

$$\dot{I}_a = \iint_A \dot{q}'' \left(\frac{T_0}{T_j} - \frac{T_0}{T_{j,\text{max}}} \right) dA \quad (13)$$

This avoidable loss is actually an exogenous irreversibility since it is not constrained by the chip but determined by heat sink design. Indeed, it is the heat sink which governs the junction temperature T_j . Therefore minimization of \dot{I}_a should be regarded as a challenge in heat sink design.

Figure 4 shows the Grassmann diagram of the heat sink cooled chip. The intrinsic irreversibility \dot{I}_i together with the maximum amount of exergy which can be passed on to the heat sink are hatched. Notice that although the intrinsic irreversibility \dot{I}_i often is the largest irreversibility in a heat sink cooled chip system, it is unavoidable. This illustrates that it is not always possible to allocate engineering efforts solely based on the absolute value of irreversibility. Furthermore one can deduce from the diagram and corresponding equations (Eq. (7) and (13)) that a reduction in junction temperature T_j reduces the irreversibility in the heat sink \dot{I}_{hs} but on the other hand increases the avoidable irreversibility in the chip \dot{I}_a . Since chip and heat sink are thermodynamically dependent one can not lower the irreversibility in one component while assuming the other won't be affected.

Design

Second Law based design endeavors to minimize the difference between reality and the corresponding exergy model. Al-

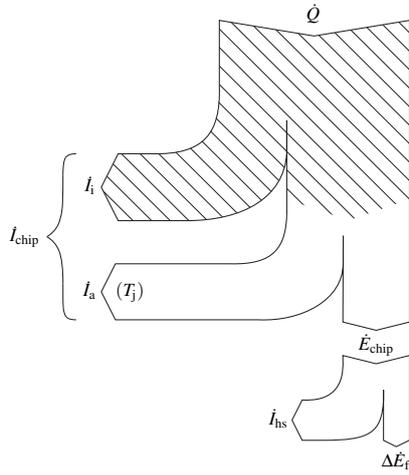


Figure 4. Grassmann diagram for a chip heat sink combination with $T_{in} = T_0$.

ternatively formulated it strives to minimize the irreversibility. Since irreversibility is proportional to entropy generation, engineering literature conveniently baptized Second Law based design as entropy generation minimization or EGM [18].

Entropy generation minimization allows to compare different interactions on a common basis [3]. This is one of the benefits associated with a Second Law based design methodology often found in literature. The Second Law reduces the number of objectives as it eliminates an ad hoc trade-off between heat transfer and fluid flow losses since a trade-off is embedded in the concept of irreversibility or entropy generation [32–34]. Yet this trade-off is based on the exergy model. How meaningful is this trade-off if applied to real applications which are irreversible?

To answer this question we examine a Brayton cycle as depicted in Fig. 5. This Brayton cycle takes a mechanical exergy input (P_C) and a thermal exergy input (\dot{Q}_{cc} at temperature T_{cc}) to generate a mechanical exergy output (P_T) while producing an exhaust flow exergy (e_6). A Brayton cycle with a topology as is presented will not use this flow exergy e_6 . Therefore the exergy of e_6 is lost. Since this flow exergy is mainly composed of thermal exergy we can conclude that due to the cycle topology the actual work potential of heat transfer irreversibility is lower than the theoretical work potential indicated by the exergy model.

The cycle turbine uses a pressure and temperature difference to generate the turbine power P_T . Divergence of the isobaric lines, the compressor pressure ratio, the heat transfer and fluid flow efficiency of the recuperator together with the isentropic efficiency of the turbine determine how P_C on one hand and \dot{Q}_{cc} on the other will be used to produce power. Since all components are irreversible, exergy as such and work potential attributed to thermal or mechanical energy specifically do not reflect the true potential of the energy streams to produce turbine power.

Previous reflections illustrate that entropy generation minimization of a heat exchanger as a component cannot provide the most optimal recuperator for a Brayton cycle. Although heat exchanger design is a trade-off between momentum losses and heat transfer enhancement it is not necessarily entropy generation that provides the optimal trade-off. Indeed, the actual work potential of energy streams is determined by irreversible components. Therefore it is different from the theoretical work potential as derived from an exergy model.

Entropy generation minimization has been applied to design

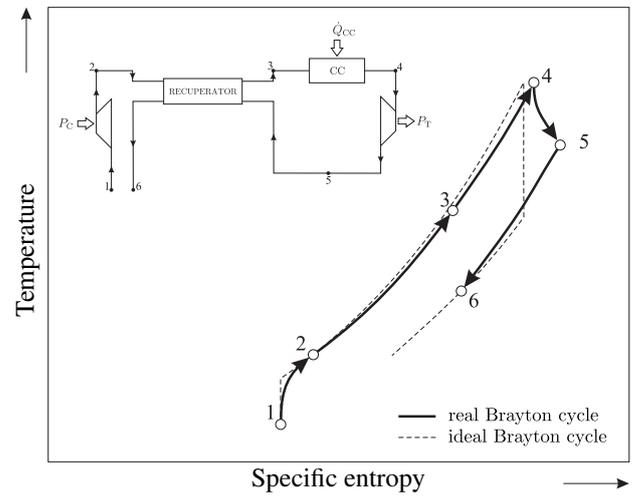


Figure 5. T-s diagram of a Brayton cycle with recuperation.

a large variety of components. Especially in the field of heat exchangers and heat sinks EGM has acquired some renown as optimization criterion since energy falls short in quantifying the performance of these components (e.g. [14, 16, 33, 35–37]). However, component optimization is not necessarily in correspondence with system optimization.

Thermodynamic optimization of a system is equivalent to a minimization of the total entropy generation \dot{S}_{gen}^{tot} which is an addition of the entropy generation in all components (n)

$$\dot{S}_{gen}^{tot} = \sum_{i=1}^n \dot{S}_{gen}^i \quad (14)$$

Minimizing \dot{S}_{gen}^{tot} is

$$\min \dot{S}_{gen}^{tot} = \min \sum_{i=1}^n \dot{S}_{gen}^i \quad (15)$$

$$\neq \sum_{i=1}^n \min \dot{S}_{gen}^i \quad (16)$$

meaning that optimized components do not necessarily result in an optimized system unless these components are thermodynamically isolated [18]. This simple mathematical reflection urges to raise a note of caution considering the application of EGM on component or on smaller scales without considering the overall system.

Entropy generation minimization assumes an invariable environment. Applying entropy generation minimization on a component or subsystem is therefore identical to casting the remaining part of the system as an invariable environment model. Such a model does not represent reality as it does not incorporate the effects of a local entropy generation minimization on another location or in another time frame.

Beyer addressed this issue already in the 70s [23, 30, 38] and also in subsequent decades similar remarks have been formulated mainly in the field of thermo-economics [25, 31, 39–42]. Entropy generation minimization has to be applied on the overall system or on independent components or subsystems. If not, the objective is different from system efficiency and therefore meaningless if thermodynamic efficiency is targeted. A system is in general more than the sum of its parts. It is a complex network of components and elements which influence each other.

CONCLUSION

If reversibility would be feasible then exergy can be conserved and the exergy model could be an accurate model. Unfortunately reversibility is unattainable and therefore the equality sign in the formulation of the Second Law is deceptive. The inequality sign of the Second Law is what reality defines. It is this inequality that represents the arrow of time and indicates direction. A direction which is incorporated in transfer and transport functions (e.g. Fourier's Law, Fick's Law).

The Second Law only enables us to pinpoint and quantify losses relative to the reversible ideal. Since reversibility is infeasible every entity under consideration creates losses. However the causes of these losses remain unknown and consequently engineering or economic efforts can not be allocated based on these losses. Minimization of all losses is equivalent to thermodynamic efficiency optimization. Entropy generation minimization is therefore an alternative cost function for First Law optimization. Anyhow it does not allow to decouple a system and design interacting components in thermodynamic isolation.

ACKNOWLEDGMENT

The work of Ruben Gielen is sponsored by the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT Vlaanderen).

Nomenclature

A	heat source base area [m ²]
C	heat transfer surface inventory [W/K]
\dot{E}	exergy [W]
E	energy [J]
e	flow exergy [J/kg]
g	gravitational acceleration [m/s ²]
h	specific enthalpy [J/kg]
\dot{I}	irreversibility [W/K]
\dot{m}	mass flow rate [kg/s]
P	power [W]
\dot{Q}	heat [W]
q''	chip heat load per source base area [W/m ²]
S	entropy [J/K]
s	specific entropy [J/kg K]
\dot{S}_{gen}	entropy generation [W/K]
T	temperature [K]
V	velocity [m/s]
\dot{W}	work [W]
z	height [m]

sub/superscripts

0	dead state
cc	combustion chamber
f	fluid
in	inlet
j	junction
max	maximum
out	outlet
tot	total

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