

## ENERGY/EXERGY BASED COST ACCOUNTING IN LARGE ECOLOGICAL-TECHNOLOGICAL ENERGY SYSTEMS

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### ABSTRACT

If a sufficiently extensive network of energy connections is considered, it is evident how all production and technological systems are directly or indirectly supported by relationships with the ecological systems of the biosphere that surrounds them. As a consequence, some of the energy flows, consumed inside production processes and technological devices, cross the limits of the control-volume and finally, in order to take into account the effects on the whole macro-system of the optimal design choices that we are looking for, the following question has to be answered:

“How much primary energy has been used by the macro-system to maintain each one of those flows at a defined value?”

In spite of some similarities, the two widely used methodologies show important differences, too, so that the answers to the previous question are often not-consistent, even if very simple systems are considered. Nevertheless, an almost complete integration among the two methodologies appears to be at hand, while all major differences can be explained, if we think about the possible behavior of the different components inside the system, instead of the axioms of previous formulations. The integrated approach is expected to enlarge the options the analyst can use to define and optimize the system and to allow the correct use of the results of both methodologies.

### INTRODUCTION

An energy system can be synthetically described as a network of energy flows, connecting some nodes (named components, or sub-systems) where different kinds of irreversible, energy conversion processes may occur. This point of view is widely adopted for analyzing either natural, biological and ecological systems, or human-made, technological production systems. In Fig.1 a combined heat and power (CHP) system is shown, while in Fig.2 a simple energy diagram is reported for a forest with natural wood production.

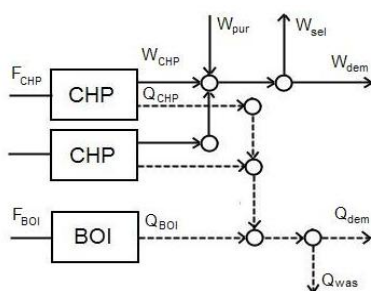


Fig.1 CHP system [1]

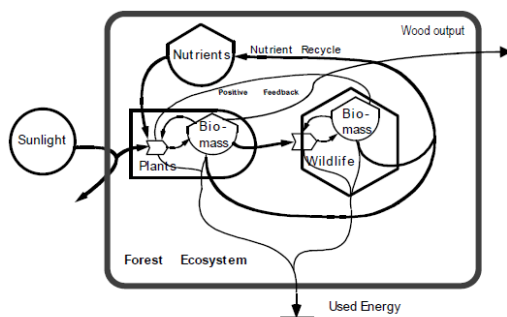


Fig.2 Emergy diagram for the wood production [2]

Indeed, ecological and technological energy systems are not disjoint sets of things, but they are strongly interconnected and interact each other, as it is especially evident when biomass-to-energy conversion systems, or bio-fuel production plants are taken into account.

Not only that, if we consider a sufficiently extensive network of energy connections, it is evident how all technological systems are directly or indirectly supported by relationships with the ecological systems of the biosphere that surrounds them. Therefore, when we focus on the design and optimization of an energy conversion system, or of a component inside a production plant, we actually define a control-volume, isolating a sub-system from a much bigger and much more complicated macro-system. As a consequence, some of the energy flows, consumed inside the sub-system, cross the limits of the control-volume and finally, in order to take into account the effects on the whole macro-system of the optimal design choices that we are looking for, the following question has to be answered:

“How much primary energy has been used by the macro-system to maintain each one of those flows at a defined value?”

Various methodologies can be found in literature to address this problem. Two of them, in particular, are widely used and defined in details. They are:

- The EMerger Analysis (EMA), defined in the ambit of the ecological modeling and ecological economics,
- The Exergy Cost Theory (ECT), defined in the ambit of energy conversion engineering.

In spite of some similarities, these two methodologies show more than a few important differences, so that the answers to the previous question are often not-consistent, even if very simple systems are considered.

The paper highlights first the similarities, avoiding confusion between exergy and exergy cost analysis, in the comparison with the EMA, and introducing the parallel concepts of transformity ( $x$ ) and unit exergy cost ( $k^*$ ), by using the derivative approach, typical of the Structural Theory of Termoeconomics. The possible interpretation of  $x$  and  $k^*$  as shadow (marginal) or average cost is also briefly discussed.

The differences arising when bifurcating flows, or recycles, are considered inside the system, are then introduced by showing some very simple cases, and an effort is done to bring the different approach about bifurcating flows to some (implicit or explicit) hypothesis on the physical behavior of the multi-product component, where the bifurcation occurs. Finally, by extending a recent result of the EMA (the Dynamic EMergy Accounting), a general formulation of cost allocation problem in case of recycling flows is obtained: in this way, the unit exergy cost ( $k^*$ ) of the ECT can be re-obtained as a particular case, with a well-defined physical meaning.

### THE SYSTEM AS A NETWORK OF ENERGY FLOWS

Let's consider a steady state operation of a multi-component energy system (notice that its Total Fuel ( $F_T$ ) could be regarded as the whole consumed energetic and/or economical resources). If the energy/exergy flows inside the network are defined in order to properly describe the productive relations among components and with the outside of the system, each component (or process) can be regarded, at local level, as an autonomous production unit, having one, or more, output flow named *Products* or *Functions* and one or more input flows, named *Fuels* or *internal resources*. As a result, a sort of *local model* of each component is isolated from the whole system thermodynamic model, while the network can be regarded as the so called Productive Structure (PS) of the system.

Each flow  $E_i$  describing a *productive relation* among components has to be defined on the basis of heat, work and mass flow rates and of thermodynamic conditions of working fluids inside the plant. From a mathematical point of view, the choice of the analytic formulation is free and is left to the Analyst. If exergy flows were used to describe the productive relations inside the system, additional information could be obtained about losses inside each control volume and about distance from reversibility of each energy conversion process [3, 4, 5, 6]. Nevertheless, the definition of a simple (e.g. linear) model can be sometimes simplified by using energy based descriptions for the productive relations ( $E_i$ ). In any case, exergy based productive relations have to be regarded as the general option [7, 8, 6].

Let's go back to the basic question: "How much primary energy has been used by the macro-system to maintain each flows of the PS at a defined value?" If the system is similar to a linear chain (like the one in Fig.3) and it is operating in stationary condition, the answer can be easily inferred. In fact, flows  $E_1$  and  $E_2$  do correspond to the primary energy  $F_1$  and  $F_2$ , respectively; flow  $E_3$  is maintained by  $F_1$ , so that its *unit energy/exergy cost* is defined as  $k^*_3 = F_1 / E_3$ , and a similar situations happens for flows  $E_4$  and  $E_5$ , too. Let's

think at the bifurcation of flow  $E_3$  as a split, without any thermodynamic transformation or process, therefore flow  $E_7$  results maintained by a fraction  $E_7 / E_3$  of flow  $F_1$  and its unit cost is defined as  $k^*_7 = (E_7 / E_3) (F_1 / E_7) = k^*_3$ . Flow  $E_8$  is maintained by the remaining part of flow  $F_1$  and by the entire flow  $F_2$ . Its unit cost is  $k^*_8 = k^*_3 (E_6 / E_8) + k^*_5 (E_5 / E_8)$ .

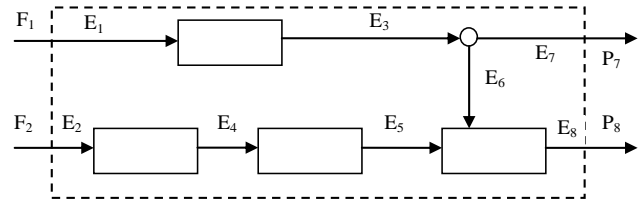


Fig. 3 A simple linear system with a split.

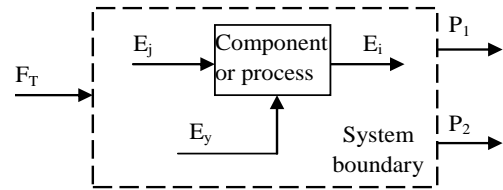


Fig. 4 A generic single product component.

The ratios like  $F_1 / E_3$  (or  $E_6 / E_8$ ) can be defined as the specific consumptions (or the partial specific consumptions) for obtaining a certain energy/exergy flow inside the system. This approach is formalized in deep detail in [4], where specific exergy consumptions are named  $\kappa_{ij}$  and a very elegant matrix formulation is introduced. In matrix form, the input/output relations for all components or processes inside the PS, as well as all unit energy/exergy costs, can be expressed as follows:

$$\mathbf{E} = {}^t\mathbf{K}\mathbf{E} + \boldsymbol{\omega} \quad (1)$$

$$\mathbf{k}^* = [\mathbf{U}_D - \mathbf{K}]^{-1} \cdot \mathbf{c}_e \quad (2)$$

where  $\mathbf{K}$  is a square matrix containing the specific energy/exergy consumptions  $\kappa_{ij}$ , while  $\boldsymbol{\omega}$  and  $\mathbf{c}_e$  are vectors containing the out coming products (required by external users) and the unit energy/exergy cost of the incoming primary energy resources, respectively. Eq.(1) is named the *Characteristic Equation* of the PS.

Notice that costs inside vector  $\mathbf{k}^*$  have to be regarded as *average costs*, because they are obtained at constant specific energy/exergy consumption (or at constant energy/exergy efficiency) of all components and processes inside the PS. Therefore, all energy/exergy units, making up a certain flow, are regarded as obtained with the same efficiency of the production chain that starts with the incoming energy resources and reaches the considered flow.

For optimization purposes, *marginal costs* may be regarded as more appropriate, but two remarks have to be taken into account:

- Dealing with large ecological-technological energy systems, the *state* of a production chain is generally supposed to be not affected by the additional production of a unit of a certain commodity (so that we can speak about the unit energy cost of electricity, natural gas, paper or corn without the need of specifying the exact production level at which that unit has been produced);

- Marginal cost cannot be simply obtained from the values of all flows in the steady state operation of the system, even if it is similar to a linear chain, but variable load operation of each component or process has to be taken into account. In addition, marginal cost can be directly calculated from the system model for those flows only corresponding to *independent* variables of the model. For dependent flows (or state variables) shadow costs only can be calculated, by introducing some additional fictitious variables into the system model.

To introduce marginal and shadow costs, let's suppose that the global model of the whole system is available. In this model each exergy / energy or cost flows in the PS can be expressed as a function of an independent variables set ( $\tau_T$ ), made with all necessary variables for describing the behavior of the specific component or process obtaining that flow as a Product.

The *local model* of a general single-product component (Fig.4) can be first formulated as shown in the left hand side below, by operating a simple variable change, i.e. by identify the Product as one of the local independent variables:

$$\begin{aligned} E_i &= f_i(\tau_i) = f_i(\tau_1, \tau_{T-1}) \Rightarrow f_i^{-1}(E_i, \tau_{T-1}) & (3) \\ E_j &= f_j(\tau_T) = f_j(f_i^{-1}(E_i, \tau_{T-1}), \tau_{T-1}) \Rightarrow E_j = f_j^+(E_i, \tau_{T-1}) \\ E_y &= f_y(\tau_T) = f_y(f_i^{-1}(E_i, \tau_{T-1}), \tau_{T-1}) \Rightarrow E_y = f_y^+(E_i, \tau_{T-1}) \end{aligned}$$

The hypothesis that a component has to have one and only one Product is widely applied in Literature. The choice of a single outgoing flow, representing the purpose, or the *Product* of the component, makes easier the economic interpretation of the PS, as seen before.

The problem of multi-product components is the key point where differences arise between ECT and EMA and will be discussed in the following. In this paragraph it has to be stressed that in a lot of cases the apparent second product is an outgoing flow from a control volume having its main product, but the second product may be regarded as a sub-product or a residue, depending on whether its production implies a reduction, or an increase of total fuel consumption, respectively, at global level. The cost formation process of by-products and residues in the ambit of the ECT is discussed in [9, 10]. The heat cogenerated by an internal combustion engine can be regarded as an example of sub-product (when it is supplied to thermal users), or of residue (when it is not).

In other cases, inside the multi-product component an internal PS could be defined, where each sub-component obtains only one product [3].

The previous local model formulation, expressing local resources as well as eventual sub-product and residue vs. the local product, can be linearized, with the aim of obtaining the Local Linear Model (LLM).

$$\begin{aligned} E_j &= \mu_{ij}(\tau_{T-1}, E_i)E_i + \theta_j(\tau_{T-1}, E_i) \\ E_y &= \mu_{iy}(\tau_{T-1}, E_i)E_i + \theta_y(\tau_{T-1}, E_i) \end{aligned} \quad (4)$$

In these relations the coefficients of the linearizations are regarded as functions of the product too. This is the general case. But, if the production level variations are restricted inside an interval where a specific linear relation is acceptable, the linearization coefficients can be regarded as

*independent* from the production level of the component. This property of the LLM is expected to be an important advantage in further optimization procedures. In fact, recollecting the LLMs of all components and nodes (junctions and branches), the minimum resource consumption for the system described by the PS, can be obtained as the solution of the following MILP problem:

$$\min. F_T = {}^t\mathbf{c}_e \cdot \mathbf{E} \quad (5)$$

$$\mathbf{E} = {}^t\mathbf{M}\mathbf{E} + \mathbf{Q} + \boldsymbol{\omega} \quad (6)$$

$$\mu_{ij} = f_{ij}(\tau_{T-1}) \quad (7)$$

$$\theta_i = g_i(\tau_{T-1}) \quad (8)$$

where  $\mathbf{M}$  is a square matrix and  $\mathbf{Q}$  is a vector containing the linearization coefficients  $\mu$  and  $\theta$ , respectively.

Let's suppose that a feasible solution were available; a general variation in the objective function  $F_T$  can be expressed rearranging the total derivative of Eq.(5), taking into account linearization coefficients behavior (Eq.(7), (8)). The procedure is very similar to that presented in [16], where additional details can be found. In this way the so called *Fuel impact formula* is obtained:

$$dF_T = \sum [ {}^t\mathbf{E} \cdot d\mathbf{M}(\tau_T) \cdot \boldsymbol{\lambda} + {}^t\boldsymbol{\lambda} \cdot d\mathbf{Q}(\tau_T) + \sum \lambda_u \cdot d\omega_u ] \quad (9)$$

$$\boldsymbol{\lambda} = [\mathbf{U}_D - \mathbf{M}(\tau_T)]^{-1} \cdot \mathbf{c}_e \quad (10)$$

Eq.(10) corresponds to the structural cost formulation, obtained by Valero - Lozano - Serra [11] through a lagrangian procedure, when vector  $\mathbf{Q}$  is equal to zero. It can be demonstrated [12] that costs  $\boldsymbol{\lambda}$  in Eq.(10) are the dual costs of the restrictions expressed by Eq.(6), therefore they have to be regarded as the marginal and shadow costs of flows  $\mathbf{E}$ .

Notice that costs  $\boldsymbol{\lambda}$  and average costs ( $\mathbf{k}^*$ ) do coincide [13, 5, 14, 15] if the LLM is replaced with the characteristic equation, i.e. if the input/output relations of each component or process are replaced by the definitions of specific energy/exergy consumptions  $k_{ij}$ :

$$\begin{aligned} E_j &= k_{ij}(\tau_{T-1}, E_i)E_i \\ E_y &= k_{iy}(\tau_{T-1}, E_i)E_i \end{aligned} \quad (11)$$

It is important to notice that this means that average costs ( $\mathbf{k}^*$ ) also can be ideally obtained through a derivative procedure; in addition they can be calculated from a single *picture* (state) of the system, while the costs  $\boldsymbol{\lambda}$  have to be obtained through an actual linearization of the energy system model.

## THE EXERGY COST THEORY AND THE EMERGY ANALYSIS

Even if the ECT terminology has been used in the previous paragraph, it can be stated that the outlined procedure for obtaining the average costs is consistent also with EMA, at least from a methodological point of view.

To recall the complete formulation of the two methodologies is behind the object of this work. Various papers about this topic may be find in the literature (see only few of them in the References: Frangopoulos [16], Odum et al. [17], Reini et al. [18], Lozano and Valero [4], von Spakovsky and Evans [20]). Nevertheless, the main point of

ECT and EMA will be re-obtained in the following, from the PS of an energy system.

Let's first focus only on the similarities between the two approaches:

- They both describe an energy system as a network of energy/exergy flows, connecting sub-systems (or components), where energy conversion (or production) processes occur.
- They both introduce junction and splitter as nodes of the network, to obtain a meaningful picture of interactions among components.
- They deal with systems in stationary state (ECT), or evaluate flows through their annual average values (EMA).
- In the network, primary energy is continuously consumed, converted and dissipated to obtain one or more product flows for the outside the system.
- They both face the fundamental question: How much of primary energy flows is directly or indirectly required for obtaining a power unit of a certain flow inside the network making up the system? The answer is the transformity ( $x_i$ ) and the unit exergy cost ( $k^*_i$ ) of the generic flow  $E_i$  for EMA and ECT, respectively.

It can be inferred that transformity and unit exergy cost have to be analogous magnitudes and the same happens for the Energy flow (Empower:  $Em_i \equiv x_i E_i$ ) and the exergy cost ( $E^*_i \equiv k^*_i E_i$ ). Energy flow and exergy cost have, in some cases, the same value too, as can be easily demonstrated for the case in Fig.3, if solar energy is regarded as the only input.

These results make evident that a strong analogy may be identified between ECT and EMA, whereas it does not make sense comparing EMA with Exergy Analysis, because the latter does not incorporate any concept of *indirect* resources consumption.

Some differences arise between the two methodologies because they suggest different solutions to the fundamental problem of multi-product component or process. In fact they both pretend to obtain the answer only on the basis of the steady state values of all flows, without the need of introducing a more detailed model of component behavior.

EMA consider two options for multi-product components:

- There are only one true product and it is then spitted in two flows, without any transformation or losses, so that the true product and the spitted flows have to be qualitatively homogeneous; this is the case of flow 3 in Fig.3.
- There are two (or more) simultaneous, heterogeneous products; in this case the whole energy (exergy cost) of the inputs is allocated on each one of the so-called co-products, without any apportionment.

The second case introduces a complication, because the co-products are the origins of different production chains that can be (partially) reunited, so that double counting of primary resources consumption has to be carefully avoided in the evaluation of the unit costs, or transformities, for the final products. This kind of double counting has naturally to be avoided also when a co-product is reunited upstream the multi-product component, i.e. when a co-product is recycled backward in its own production chain. It is surprising to notice that the same *not-double-counting* rule is applied to all recycling flows, even if they come from a split, where the

energy accounting methodology has not introduced any duplication of the primary energy resources!

The EMA methodology is often presented as a set of axiomatic rules, that can be summarized in four points (Brown and Herendeen, [21]):

- 1) All source energy to a process is assigned to the processes' output (or outputs).
- 2) By-products (multi-products) from a process have the total energy assigned to each pathway.
- 3) When a pathway splits, the two "legs" have the same transformity.
- 4) Energy cannot be counted twice: (a) Energy in feedbacks cannot be double counted; (b) by-products, when reunited, cannot be added to equal a sum greater than the source of energy from which they were derived.

If a sub-system has two product flows, the original ECT postulates one of the following three cases:

- the exergy cost of the two products is the same; this case corresponds to the first option of EMA.
- the exergy cost of one of the two products is externally fixed; this case has been afterwards developed into the residue and sub-product concepts [22].
- inside the multi-product component an internal PS could be defined, where each sub-component obtains only one product [18].

It has to be pointed out that the last hypothesis brings to a generalization of the average cost definition. In fact, the matrix formulation (Eq.(2)) allows average costs to be calculated also in cases where they cannot be inferred from the conservative cost balance of each component.

The characteristic equations for the last case and for a residue and a sub-product accompanying the main product are shown in Fig.5.

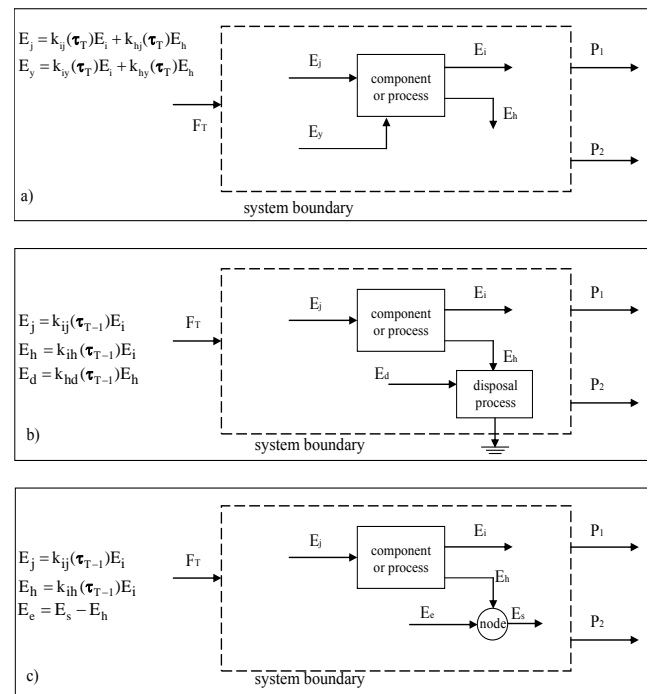


Fig.5 Characteristic Equations and PS for a multi-product component (a), a component with a residue (b) or a sub-product (c), agreeing with the ECT.

The additional main differences between the two methodologies are summarized and in the following:

- The limit of the system is not the same. In fact ECT is typically applied to power plants (or other energy conversion systems), having a fossil fuel as the main input and electric and/or thermal power as output. EMA include inside the analyzed system also the ecological sub-system of the biosphere and goods and services from the market, directly or indirectly required for operating the system.
- EMA measures energy flows through their energy content, while ECT measures energy flows through their exergy content, evaluated with respect of a proper set of ambient conditions.

The first point is not a real problem. In fact, the system limits usually defined by the ECT could be extended, up to coincide with those of EMA. In any case, the exergy input consumed by the ecological sub-system of the biosphere, as well as the exergy required to make available goods and services in the market, could be introduced into the analysis through a proper set of additional exergy costs for the system. The problem lies in the practical evaluation of these additional exergy costs, rather than in some mathematical limitation of the methodology. In this matter, the large experience of EMA could be of help to extend the application of the energy/exergy based cost accounting methodology.

The difference in the second point is vanishing in the recent years. In fact the idea of measuring energy flows through their exergy content is now widely accepted and practiced in the field of EMA (Odum, [23], Ulgiati and Brown, [24]). In addition, it could be noted that energy and exergy based cost accounting actually provide different results only if some flow splits (like flow 3 in Fig.3). Otherwise, in spite of being the unit costs different, the cost flows ( $E^*$ , or  $E_m$ ) are the same.

Thus, the different hypotheses, formulated to deal with bifurcation and recycling of flows, have to be regarded as the main impediment towards a unified formulation of the two methodologies. Notice that analogous differences can be found when comparing EMA with Embodied energy Analysis (Brown and Herendeen, [21]).

## RECENT DEVELOPMENTS

In recent years, the effort to re-think the fundamental background of EMA and the aim to enlarge its application field, are bringing inside the EMA some new contributions that could bring the two methodologies closer [25, 26, 27].

Yang et al. [25] proposed a new EMA method for waste treatment, reuse and recycle that is very similar to the sub-product and residue concepts defined in the ambit of ECT and briefly summarized in previous Fig. 5.

Reini and Valero [22, 10] have suggested how a cost allocation criteria, very similar to the case of co-product in the EMA, can be obtained in the ambit of ECT. In fact this is the result when there are two (or more) products, but only one degree of freedom is available for the multi-product component.

In this case, only one of the two products may be identified as the component degree of freedom in the LM, so that a LM similar to that shown inside Fig.5.a cannot represent the *actual behavior* of the multi-product

component, because in that case at least two, independent degrees of freedom are *implicitly* required to allow that one product could be modified independently by the other.

In other words, one product only may be put in relation with the single degree of freedom, while the second one has to be regarded as a dependent flow of the LM.

If the PS does not allow the second product to be identified as a sub-product, or a residue, the result obtained by applying Eq.(2) show a unit cost equal to zero for the second product, like it was a not required output, or a residue without any cost for disposal. To obtain the not-null value of its transformity, a complication has to be accepted: The roles of the dependent/independent flow have to be exchanged and Eq.(2) has to be applied a second time. This two-step procedure allows obtaining a couple of cost ( $k^*$ ), consistent with the co-product concept of EMA. Notice that the simultaneous production of two flows of different nature, where each one cannot be obtained without the other, has always been used in the emergy Literature to support its co-product concept and its peculiar, not-conservative, cost allocation rule. Moreover, it has to be pointed out that co-products are quite common inside living energy systems, whereas sub-product and residue cannot be easily identified in this kind of systems; the opposite happens dealing with the technological energy conversion systems, where the ECT has been generally applied.

To come to the problem of recycling flows, Tilley and Brown [26, 27] have developed an approach to deal with those process in which the recycle of material is present inside the process itself. This kind of approach (named Dynamic Emergy Accounting, DEA) provides for explicitly taking into account the dynamic characteristic of the components inside the system, not only the stationary conditions, which are supposed to be reached at the end of a transient period.

At a generic instant, the recycled flow is separated from the main product and is sent to a buffer, where it is cumulated and lies in wait to be reused within the productive process. At the equilibrium, the material quantity inside the buffer remains constant, therefore the situation is the same of the steady state operation of the system without buffer.

In reality the transient period has an impact on the transformities at the equilibrium, in fact, the recycled flow has its own emergy value which is regarded as a productive factor in the upstream production chain. This is different from what traditionally happens in the EMA, in which the not-double-counting rule applies to all recycling flows, without regarding if they come from splits or co-products.

It follows that the transformity of the material recycled flow do coincide with that of the material inside the buffer: therefore, the transformity value of the material recycled flow becomes an input datum, that must be known prior by the analyst.

Moreover, in the DEA approach, a sort of *emergy ring* results made up by the recycled flow, together with a fraction of its production chain, so that the emergy of some flows in that chain may be greater than the total emergy input of the system.

## TOWARD A UNIFIED APPROACH

By extending the result of the DEA, a general formulation of cost allocation problem in case of recycling flows can be





