CONCEPTUAL ANALYSIS OF THE ENTROPY PRINCIPLE IN CONTINUUM PHYSICS: AN OVERVIEW

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EXTENDED ABSTRACT

In continuum physics the entropy constitutive principle offers a valuable help in modeling material properties. It has been proposed by Coleman and Noll [1] within the frame of Rational Thermodynamics (RT) [2], and asserts that:

The constitutive equations, which characterize the material properties of continuous media, must be assigned in such a way that second law of thermodynamics is satisfied along arbitrary thermodynamic processes.

These authors also provided a rigorous mathematical procedure (the Coleman-Noll procedure) to exploit the principle. Afterwards, Liu [3], developed an alternative and elegant method of exploitation, based on the Lagrange multipliers, known as Liu procedure. Hauser and Kirchner [4], recognized that the Liu procedure constitutes a special case of a more general result on linear programming [5]. In Thermodynamics of Irreversible Processes (TIP) [2], de Groot and Mazur [6] and Gyarmati [7] exploited it by a phenomenological method, which regards the entropy production as a bilinear function of thermodynamical forces and fluxes. A more rigorous formulation of this procedure has been achieved by Ván et al. [8; 9].

A new perspective has been open by Müller and Ruggeri [10], who applied the principle to determine the main field which renders the system of field equations of Rational Extended Thermodynamics (RET) [2], symmetric hyperbolic, and showed how the Lagrange multipliers provide this field [11; 12]. In Extended Irreversible Thermodynamics (EIT) [2], Jou, Lebon and Casas-Vazques applied the principle to obtain nonlocal extensions of classical transport equations [13; 14; 15].

In order to derive the entropy principle by general physical laws, Muschik and Ehrentraut [16] proposed the following amendment to the classical second law:

The curves representing quasi-static processes in the state space are all contained in the equilibrium subspace.

As a consequence of this statement, they proved that second law of thermodynamics necessarily restricts the constitutive equations and not the thermodynamic processes. In this way, the classical Coleman-Noll approach follows by a more general assumption through a rigorous proof.

On the other hand, Ruggeri [12; 17], on the example of the Lax conditions for shock wave propagation in perfect fluids, observed that for weak solutions the entropy principle selects the thermodynamical processes instead of restricting the constitutive equations. Notwithstanding, in some recent articles [18; 19] it is proved that the result by Muschik and Ehrentraut may be extended to non-regular processes and generalized exploitation procedures [20], so that this problem deserves a more deep analysis.

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