DOES MINIMUM ENTROPY GENERATION RATE CORRESPOND TO MAXIMUM POWER OR OTHER OBJECTIVES ?

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ABSTRACT

In a recent book, A. Bejan has reconsidered the important problem of maximization of power with heat engine models associated to heat transfer irreversibilities (endoreversible models).
He explains that for these models maximum power is equivalent to minimum entropy generation rate (corresponding to Gouy-Stodola theorem). He also stresses that the method is well known in the engineering literature [4], if new one for physicist [5, 6, 7]. More precisely, he points out, that in these last papers [5, 6, 7], maximum power and minimum entropy generation rate are two distinct optimization criteria for power plants.
An example was given in [2] for coincidence of these two objectives. We propose here to reconsider these two approaches ant try to entlighten on simple Chambadal power plant model, but irreversible one, the conditions of equivalence between the two objectives, maximum of power, and minimum of entropy generation.
influence of various system configurations are analyzed. Consequences are discussed, and generalization of the proposed method allows to clarify the subject and to precise the equivalence conditions or not regarding other important objectives.

1 INTRODUCTION

Since the eighties, optimization of energy systems, mainly engine and reverse cycle machines, has been reconsidered starting with the paper of F. Curzon and L. Ahlborn [1].

In a recent book [2], A. BEJAN has reconsidered the important problem of maximization of power with heat engine models associated to heat transfer irreversibilities (endoreversible models).

He explains that for these models maximum power is equivalent to minimum entropy generation rate (corresponding to GOUY-STODOLA) theorem [3]. He also stresses that the method is well know in the engineering literature [4], if new one for physicist [5, 6, 7].

More precisely he points out, that in these last papers [5, 6, 7], maximum power and minimum entropy generation rate are two distinct optimization criteria for power plants. An example was given in [2] for coincidence of these two objectives.

We propose here to reconsider these two approaches and try to enlighten on simple model of power plants, but irreversible one, the conditions of equivalence between the two objectives, maximum of power, and minimum of entropy generation.

Influence of various system configurations are analyzed. Consequences are discussed, and generalization of the proposed method allows to clarify the subject and to precise the equivalence conditions regarding other important objectives. The results illustrated here from a simple pedagogical point of view, can be completed regarding recent published papers [8 - 10]. Paper [8] is a general review, that can be particularized to reverse cycle machines [9, 10]. Reference [11] reports on the energy and exergy optimization of combined heat and power systems. Reference [12] considers two examples of Exergy Optimization too including "Thermofrigopump".

The two papers [13, 14] are more fundamental and are related to new upperbounds of what we named Optimal Thermodynamics, as well as on reconsideration of criteria in order to optimize irreversible thermomechanical heat systems.

2 CHAMBADAL MODEL OF POWER PLANT

This model from 1957 is the first one proposed, and it uses a sensible heat source. This source is a finite size one, due to the fact that heat is transferred through an imposed mass flux $\dot{m}_H$, with a constant calorific C_{PH} value, so that :

$$\dot{C}_H = \dot{m}_H \, C_{PH} \tag{1}$$

Regarding figure 1 and the converter, we use thermodynamics convention $\left(\dot{W} < 0 \,;\, \dot{Q}_{HC} > 0\right)$. The studied case is focused on steady states.

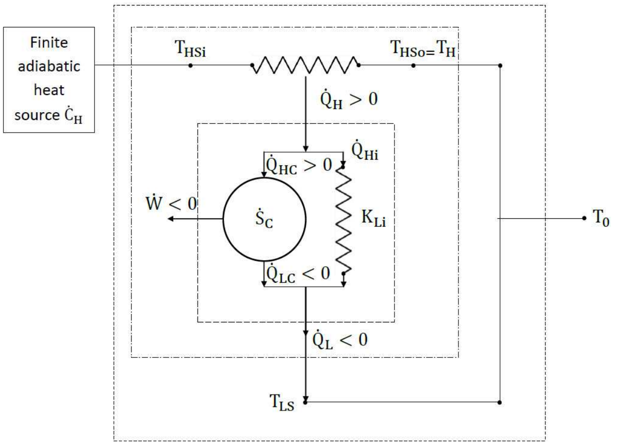

Figure 1. Chambadal power plant model

2.1 Characterization of the converter

The method of heat transfer used for heat exchangers HEX is the $(\varepsilon, \mathrm{NTU})$ method, in the reported cases. The proposed model is an extended one of the model reported by A. Bejan [2], but with internal irreversibilities of the converter.

These irreversibilities are mainly represented by the created dissipation rates inside the converter $\dot{S}_C$. This rate is the sum of all dissipation rates appearing in the converter. For Novikov, it is only related to expander irreversibilities. More generally, the prevailing irreversibilities are related to mechanical losses in the converter (associated to fluid flow and solid friction). For others it is related to heat short circuit between hot and cold side of the converter. If $\dot{Q}_{Hi}$ represents this rate, for a linear law it comes :

$$\dot{Q}_{Hi} = L_{li}(T_H - T_{LS}) \tag{2}$$

The corresponding created entropy rate is :

$$\dot{S}_{Hi} = K_{li}\frac{(T_H - T_{LS})^2}{T_H T_{LS}} \tag{3}$$

It is to be noted that $\dot{S}_{Hi}$ depends on T_H and T_{LS}

The energy and entropy balances of the converter are :

$$\dot{W} + \dot{Q}_{HC} + \dot{Q}_{LC} = 0 \tag{4}$$

$$\frac{\dot{Q}_{HC}}{T_H} + \frac{\dot{Q}_{LC}}{T_{LS}} + \dot{S}_C = 0 \tag{5}$$

with $\dot{Q}_{HC} = \dot{Q}_H - \dot{Q}_{Hi}$ (6)

$\dot{Q}_{LC} = \dot{Q}_L - \dot{Q}_{Hi}$ (7)

Accordingly to Chambadal hypothesis, we suppose perfect heat transfer at the source side such that :

$$\dot{Q}_H = \dot{C}_H(T_{Hsi} - T_H) \tag{8}$$

It means that $T_{HS0} = T_H$, $\varepsilon = 1$ (consequently hot side heat transfer area becomes infinite). This equilibrium hypothesis is adopted for simplicity and pedagogical purpose.

Combining (4 to 8) it is easy to get the general relation of the pover of the plant to maximize :

$$MAX\left(-\dot{W}\right) = MAX\left[\left(1 - \frac{T_{LS}}{T_H}\right)\left(\dot{C}_H(T_{SHi} - T_H) - \dot{Q}_{Hi}\right) - T_{LS}\dot{S}_C\right] \tag{9}$$

It is to be precised here that $\dot{S}_C$ could depend on the temperature difference accross the converter (as $\dot{S}_{Hi}$,(3)), so that we note it $\dot{S}_C(T_H)$.

$MAX\left(-\dot{W}\right)$ is obtained for a T_H variable value satisfying :

$$\frac{1}{TH^2}\left(\dot{C}_H T_{LS}T_{HSi} + K_{li}T_{LS}^2\right) - T_{LS}\frac{\partial \dot{S}c}{\partial T_H} - \left(\dot{C}_H + K_{li}\right) = 0 \tag{10}$$

The same methodology is applied to the total internal entropy created inside the converter $\dot{S}_i$, according to :

$$\frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}_L}{T_{LS}} + \dot{S}_i = 0 \tag{11}$$

By combination of (5-7) with (11) it comes :

$$\dot{S}_i = k_{li}(T_H - T_{LS})\left(\frac{1}{T_{LS}} - \frac{1}{T_H}\right) + \dot{S}_C(T_H) \tag{12}$$

The minimum of $\dot{S}_i$, min ($\dot{S}_i$), must consequently satisfy :

$$K_{li}\left(\frac{1}{T_{LS}} - \frac{T_{LS}}{T_H^2}\right) + \frac{\partial \dot{S}_C}{\partial T_H} = 0 \tag{13}$$

The only physical solution of equation (13) is $T_H = T_{LS}$, and $\dot{S}_C$ constant. It means that the converter does not deliver power, and $\dot{S}_C$ must be nul. Consequently minimum entropy created in the converter does not provide maximum power, obtained through condition (10).

If $\dot{S}_C$ is a constant (or for the endoreversible case), the following optimal temperature condition T_H^* is obtained regarding the maximum of power :

$$T_H^* = \sqrt{T_{LS} \frac{\dot{C}_H T_{HSi} + K_{li} T_{LS}}{\dot{C}_H + K_{li}}} \qquad (14)$$

We retrieve the nice radical, if $K_{li} = 0$

2.2 Characterization of the system

As can be seen on Figure 1, the system consist of the converter, and the hot heat exchanger (boiler in the case of the power plant). The external fluid is the flue gas, whose energy comes from external adiabatic hot heat source (nuclear ; combustion ; solar). The sensible heat of the flue gas is partly transferred to the converter as was indicated before (section 2.1).

The energy and entropy balances of the system are :

$$\dot{W} + \dot{Q}_H + \dot{Q}_L = 0 \qquad (15)$$

$$\dot{C}_H \ln \frac{T_{HSi}}{T_H} + \frac{\dot{Q}_L}{T_{LS}} + \dot{S}_S = 0 \qquad (16)$$

Relation (15) is identical to relation (4). Equation (5) remains too as a constraint. So $MAX(-\dot{W})$ does not change, nor the first law efficiency (or others) for the system.

Simply the created entropy inside the system comprises now the entropy due to the heat transfer between the source hot fluid and the converter. Combining (5-11) with (16) it comes :

$$\dot{S}_S = \dot{C}_H \left(\frac{T_{HSi} - T_H}{T_H} - \ln \frac{T_{HSi}}{T_H} \right) + K_{li} (T_H - T_{LS}) \left(\frac{1}{T_{LS}} - \frac{1}{T_H} \right) + \dot{S}_C \qquad (17)$$

min $(\dot{S}_S)$ is obtained for T_H satisfying :

$$-\frac{\dot{C}_H T_{HSi} + K_{li} T_{LS}}{T_H^2} + \frac{\dot{C}_H}{T_H} + \frac{\partial \dot{S}_C}{\partial T_H} + \frac{Kli}{T_{LS}} = 0 \qquad (18)$$

This equation differs from (10). Consequently the maximum of power of the system does not correspond to min of entropy generated inside the system.

It is relatively easy to show, that this result is not affected, if we add direct external linear heat loss between the hot source and heat sink of the system.

2.3 Characterization of the system in the environnement

Referring again to Figure 1, and supposing adiabaticity between the finite heat source and the environment, it appears a transiting heat rate $\dot{Q}_{Ho}$ such that :

$$\dot{Q}_{H0} = \dot{C}_H (T_H - T_0) \qquad (19)$$

This heat rate could be used for combined heat and power (CHP), eventually in organic Rankine Cycle (ORC) or others [12].

It corresponds the following exergy rate :

$$\dot{E}_{X0} = \dot{C}_H \left[(T_H - T_0) - T_0 \ln \frac{T_H}{T_0} \right] \qquad (20)$$

<u>Hot fluid as reference.</u>

The energy and entropy balances of the system in the environment becomes from the hot fluid point of view :

$$\dot{W} + \dot{Q}_{HS} + \dot{Q}_{LS} = 0 \qquad (21)$$

$$\dot{C}_H \ln \frac{T_{HSi}}{T_0} + \frac{\dot{Q}_{LS}}{T_0} + \dot{S}_0 = 0 \qquad (22)$$

with $\dot{Q}_{HS} = \dot{Q}_H + \dot{Q}_{H0}$

$$\dot{Q}_{LS} = \dot{Q}_L - \dot{Q}_{H0}$$

We renew here that if $\dot{Q}_{H0}$ could be valorized, it is the same for $\dot{Q}_L$, if T_{LS} differs from T_0. $\dot{Q}_{HS}$ corresponds to the imposed heat rate consumption (constraint related to the fluid mass rate and T_{HSi}, T_0).

The $MAX(-\dot{W})$ is always furnished by the equation corresponding to (10), but the value of the efficiency at maximum power differs, due to change in energy expanses $\dot{Q}_{HS}$.

To simplify, we suppose that T_{LS} is identical to T_0, the ambient temperature. This hypothesis remains consistent with the Chambadal model of power plant.

Using (5-8, 22) it comes after some calculations the condition for min $\dot{S}_0$:

$$\frac{1}{T_H^2} \left[\dot{C}_H T_0 T_{HSi} + K_{li} T_0^2 \right] - T_0 \frac{\partial \dot{S}_C (T_H)}{\partial T_H} - \left(\dot{C}_H + K_{li} \right) = 0 \qquad (23)$$

This equation is identical to (10) due to the fact that TLS = T0. In that case $MAX(-\dot{W})$ corresponds to min ($\dot{S}_0$), created entropy rate for the system hot fluid in the environment.

Heat source as reference

The energy and entropy balance of the system in contact with the heat source at T_{Hsi} (thermostat) and the ambient cold sink at $T_{LS} = T_0$ becomes now (21) and (24).

$$\frac{\dot{Q}_{HS}}{T_{HSi}} + \frac{\dot{Q}_{LS}}{T_0} + \dot{S}_{00} = 0 \qquad (24)$$

We have always

$$\dot{Q}_{HS} = \dot{Q}_H + \dot{Q}_{H0}$$

$$\dot{Q}_{LS} = \dot{Q}_L - \dot{Q}_{H0}$$

Using (5-8, 24) it comes after calculations, the condition for min ($\dot{S}_{00}$). We obtain again the equation (23), identical to (10) with $T_{LS} = T_0$. $MAX(-\dot{W})$ corresponds too to min ($\dot{S}_{00}$), created entropy rate between the thermostat T_{HSi} , necessary to produce the hot fluid, and the environment at T_0.

3 DISCUSSION AND CONCLUSIONS

This paper has reconsidered the optimizations regarding maximum power of a thermomechanical engine, and minimum entropy generation rate for steady state configurations.

The convenient model of Chambadal power plant has been choosen, but extended, taking particularly account of internal irreversibilities of the converter (Carnot engine).

It has been proved that these internal irreversibilities depend on T_H temperature. An example has been developed regarding heat losses between hot (T_H) and cold (T_{LS}) side of the converter.

3.1 Comparison of $MAX(-\dot{W})$ condition with min $\dot{S}_i$, total internal entropy created inside the converter

These two conditions differ. Min ($\dot{S}_i$) occurs for a plant that does not deliver power ($T_H = T_{LS}$).

The maximum power condition leads to T_H* for the endoreversible converter

$$T_H{*} = \sqrt{T_{LS}\,\frac{\dot{C}_H\,T_{HSi} + K_{li}T_{LS}}{\dot{C}_H + K_{li}}}$$

This value gives a generalized form of the nice radical.

3.2 Comparison of $MAX(-\dot{W})$ condition with min ($\dot{S}_S$), total entropy created within the system

The condition for min ($\dot{S}_S$) (18) differs from the one corresponding to MAX($-\dot{W}$) (10), even for endoreversible system, where min ($\dot{S}_S$) corresponds to the thermodynamic equilibrium situation ($T_H = T_{HSi}$).

3.3 Comparison of $MAX(-\dot{W})$ condition with the minimum of total entropy created for the system in the environment

In that case, it appears a transiting heat rate $\dot{Q}_{H0}$. This heat rate, as the one rejected at the cold sink ($\dot{Q}_L$ at T_{LS}) is supposed degraded, as done by A. Bejan. But in fact, it could be valorized (through CHP system, ORC system or others).

If not, it contributes effectively to entropy generation for both heat fluxes ($\dot{Q}_{H0}$ from T_H, to T_0 ; $\dot{Q}_L$ from T_{LS}, to T_0).

Reported calculations are relative to the common case where T_{LS} equal T_0.

It has been shown that, whatever is the reference (hot fluid, or heat source), $MAX(-\dot{W})$ is associated to the min of generated entropy rate. It comes for the endoreversible converter, the same relation as in a section (3.1) with $T_{LS} = T_0$.

3.4 Conclusions

Maximization of power, or minimization of entropy generation are equivalent, if we consider the system in his environment. This is confirmation of the Gouy Stodola theorem. But it supposes that all rejected heat are not valuable. This must be reconsidered and is an actual challenge [11, 12].

Regarding the converter and the system in itself, the two objectives are not identical.

If maximization of power is a clear objective function, regarding entropy is not so easy. We have shown here that results differ, if considering entropy of the converter, or system in the environment (including hot fluid, or hot source).

Preceding results obtained in the literature have been precized and extended, clarifying the existing controversy. These results remains to combine, to existing ones [8, 14].

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